

The Schrödinger wave equation

What it is

The Schrödinger equation is a fundamental equation in quantum mechanics that describes how the quantum state of a physical system changes over time. It was developed by Erwin Schrödinger in 1926 based on the conservation of energy and the de Broglie hypothesis of matter waves.

Forms of the equation

There are two main forms of the equation:

- **Time-independent Schrödinger equation:** This form is used for systems in a stationary state where the potential energy, V , does not depend on time.

$$-\frac{\hbar^2}{2m} \nabla^2 \psi + V\psi = E\psi$$

Where:

- ψ (psi) is the time-independent wave function.
 - $\hbar$ is the reduced Planck constant ($\hbar = h/2\pi$).
 - m is the mass of the particle.
 - ∇^2 is the Laplacian operator.
 - V is the potential energy of the particle.
 - E is the total energy of the particle.
- **Time-dependent Schrödinger equation:** This more general form describes how the wave function evolves over time.

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi + V\psi = H\psi$$

Where:

- H is the Hamiltonian operator, which represents the total energy (kinetic + potential) of the system.
- $\psi(\mathbf{r}, t)$ is the time-dependent wave function. 

Significance of the wave function (ψ)

What it represents

The wave function, denoted by the Greek letter psi (ψ), is a mathematical function that completely describes the quantum state of a particle or a system. It is a complex-valued function of the particle's position and time.

Physical interpretation

The wave function itself has no direct physical meaning. It is not an observable quantity. However, the square of its magnitude, or modulus squared ($|\psi|^2$), is physically meaningful.

Probability density: $|\psi|^2$ gives the probability density of finding the particle at a specific point in space and time. For example, in chemistry, this is often visualized as an "electron cloud" or an atomic orbital.

Key properties of the wave function

For a solution to the Schrödinger equation to be physically realistic, the wave function must meet certain conditions:

- **Continuous:** The wave function and its first derivative must be continuous.
- **Single-valued:** There must be only one value of ψ for any given point in space at a given time.
- **Finite:** The wave function must have a finite value everywhere to be physically meaningful.
- **Normalized:** The total probability of finding the particle somewhere in all of space must be equal to 1. Mathematically, this means the integral of $|\psi|^2$ over all space must equal 1.

What it helps determine

Solving the Schrödinger equation to find the wave function for a given system allows us to determine several quantized properties of the particle, such as:

- **Allowed energy levels:** The solutions provide the possible discrete energy values that the particle can have.
- **Momentum:** All information about the particle's momentum is contained within the wave function.
- **Quantum numbers:** For atoms, the solutions naturally give rise to the principal, azimuthal, and magnetic quantum numbers.

Example: Particle in a box

This simple example illustrates the core concepts.

1. An electron is confined to a one-dimensional box.

2. The Schrödinger equation is applied with boundary conditions (the probability of finding the electron outside the box is zero).
3. The solutions reveal that only certain discrete energy levels and wave functions are allowed for the electron.

PRACTICE QUESTIONS

1. Write the Schrodinger wave equation.

2. What does Schrodinger's wave equation tell us?

3. What is ψ in Schrodinger's equation?

4. What is the difference between the time-dependent and time-independent Schrodinger equations?

5. What are the applications of Schrodinger's equation in real life?

6. Significance of the wave function (ψ) and ψ^2 in Schrodinger's equation.

7. What is the full Schrodinger equation?
